E - 3828

M. A. / M. Sc. (Final) EXAMINATION, 2021

MATHEMATICS

(Optional)

Paper Third (i)

GRAPH THEORY

Time : Three Hours] [Maximum Marks : 100

Note :-Attempt any two parts from each question. All questions carry equal marks.

1. a. Let G be a labeled graph with adjacency matrix A. Then prove that the i, j entry of A^n is the number of walks of length n form V_i to V_j.

   b. Let G be a connected labeled graph with adjacency matrix A. Then prove that all cofactors of the matrix M are equal and their common value is the number of spanning trees of G.

   c. If G has incidence matrix B and cycle matrix C, then prove that

$$CB^T \equiv 0 \pmod 2$$

2. a. Prove that the following are equivalent for any graph G :

   i. G has a line –core.

   ii. G has an external minimum point cover.

   iii. Every minimum point cover for G is external

   b. Prove that a graph is bicolorable *iff* it has no odd cycles.

   c. State and prove Norman Rabin theorem.

3. a. Prove that for any graph G with six points, G or $\overline{G}$ contains a triangle.

   b. Prove that the maximum number of lines among all p-point graphs with no triangles is $[p^2/4]$.

   c. Prove that a graph G is a clique graph iff it contains a family F of complete subgraphs, whose union is G, such that whenever every pair of such complete graphs in some subfamily $F^/$ have a nonempty intersection, the intersection of all the members of $F^/$ is not empty.

4. a. Prove that the line-graph and the point-group of a graph G are isomorphic iff G has at most one isolated point and K_2 is not a components of G.

   b. For any finite abstract group F, prove that there exists a graph G such the ΓG and F are isomorphic.

   c. Explain graph enumeration and deduce the counting polynomial for graph for p points.

5. a. Prove that a digraph is strong *iff* if has a spanning closed walk.

   b. Prove that an acyclic digraph D has at least one point of indegree zero.

   c. State and prove Menger's theorem.

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